

DOI 10.1007/s10891-017-1702-y

*Journal of Engineering Physics and Thermophysics*, Vol. 90, No. 6, November, 2017

## HYDROGASDYNAMICS IN TECHNOLOGICAL PROCESSES

## DYNAMICS OF WAVES IN MULTIFRACTIONAL BUBBLE LIQUIDS

D. A. Gubaidullin and R. N. Gafiyatov

UDC 532.529:534.2

*A study has been made of pulsed waves in mixtures of a liquid and a dispersed phase consisting of vapor–gas and gas bubbles that differ in radii and thermophysical properties. A system of differential equations has been proposed for description of the motion of such a mixture and a dispersion relation has been derived for it. A comparison has been made of the dynamics of acoustic waves in mixtures of water with vapor–air bubbles, bubbles of carbon dioxide with steam, and helium bubbles, and also in monodisperse mixtures of water with bubbles of one gas. In the investigations, the qualitative composition of the dispersed phase varied due to the buildup in the volume content of the bubbles of one fraction and the relevant decrease in the content of the bubbles of the other fraction, with the total volume content of the bubbles being constant.*

**Keywords:** *acoustic waves, bubble liquid, dispersion relation, heat and mass transfer.*

**Introduction.** Studies of the wave dynamics of dispersive media are of great current interest. Numerous works on the acoustics of bubble liquids are devoted to a theoretical study of the propagation of harmonic disturbances in monodisperse mixtures. Various problems of the acoustics of mixtures of liquids and gas or vapor bubbles have been considered in the existing monographs [1, 2]. The main features of two-phase media of bubble structure have been described in [3]; works on the propagation of waves in liquids with bubbles of constant mass and on the wave dynamics of liquids containing vapor or soluble-gas bubbles have also been reviewed. A model of propagation of plane small-amplitude pressure waves in a mixture of a liquid and gas bubbles has been presented in [4] and its good performance at subresonant frequencies as applied to mixtures with a volume content of the dispersed phase of 1–2% has been demonstrated. In [5], a study has been made of the propagation of acoustic waves in two-fraction mixtures of a liquid and vapor–gas and gas bubbles of varying size and composition with phase transitions. In [6, 7], consideration has been given to the propagation of acoustic waves in two-fraction mixtures of a liquid and polydisperse gas bubbles of varying composition; a comparison has been made of the theory and the existing experimental data. In [8], a study has been made of the propagation of acoustic waves in multifractional mixtures of a liquid and vapor–gas and gas bubbles of varying size and composition with phase transformations. The present work seeks to study the evolution of pulsed pressure waves in multifractional bubble liquids.

**Basic Equations.** Consideration is given to the plane one-dimensional motion of a multifractional bubble liquid in an acoustic field in the case where some fractions of the gas bubbles contain vapor and are involved in phase transformations, whereas others consist of insoluble gas. Here, the bubbles of each fraction have dimensions different from the dimensions of the bubbles of the remaining fractions. The gas of which the bubbles of each fraction consist differs in thermophysical properties from the gases and bubbles of the other fractions. The volume contents of the bubbles of each fraction  $\alpha_{2j}$  and of the liquid  $\alpha_1$  are determined as

$$\alpha_1 + \sum_{j=1}^N \alpha_j + \sum_{i=1}^M \alpha_i = 1, \quad \alpha_j = \frac{4}{3} \pi a_j^3 n_j, \quad \alpha_i = \frac{4}{3} \pi a_i^3 n_i, \quad j = \overline{1, N}, \quad i = \overline{1, M},$$

and the reduced densities of the carrier and dispersed phases, from the relations

$$\rho_1 = \rho_1^{\text{tr}} \alpha_1, \quad \rho_j = \rho_j + \rho_{vj} = \rho_j^{\text{tr}} \alpha_j + \rho_v^{\text{tr}} \alpha_j, \quad \rho_i = \rho_i^{\text{tr}} \alpha_i, \quad j = \overline{1, N}, \quad i = \overline{1, M}.$$

---

Institute of Mechanics and Mechanical Engineering, Kazan Scientific Center of the Russian Academy of Sciences, 2/31 Lobachevskii Str., Kazan, 420111, Russia; Kazan (Volga Region) Federal University, 18 Kremlevskaya Str., Kazan, 420008, Russia; email: [gubaidullin@imm.knc.ru](mailto:gubaidullin@imm.knc.ru). Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 90, No. 6, pp. 1506–1511, November–December, 2017. Original article submitted March 31, 2016.